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ON A SUFFICIENT ENCOUNTER CONDITION IN A DIFFERENTIAL GAME

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S. I. TARLINSKII
(Sverdlovsk)

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We consider a position differential game of encounter with a target set at a specified instant. We derive one sufficient condition by whose fulfillment the pursuer ensures himself a definite qualitative result of the game. The construction of the first player's strategy is based on the program construction introduced in [1 – 3]. The results abut the investigations in [1 – 5].

1. We consider a conflict-controlled system described by the vector differential equation

$$\begin{gathered} dx/dt = A(t)x + f(t, u, v) \\ x[t_0] = x_0, \quad u \in P, \quad v \in Q \end{gathered} \tag{1.1}$$

Here $f(t, u, v)$ is a continuous n-dimensional vector-valued function, u and v are the player's controls, P and Q are compacta in appropriate vector spaces. By $\{x\}_m$ we denote the vector composed from the first m $(m \leqslant n)$ coordinates of vector x. By the problem's hypothesis a convex bounded closed set M is given in the space $\{x\}_m$. The first player, directing the choice of control u, strives to encounter this set by an instant ϑ known in advance. The second player (v) obstructs this.

Let us refine the problem statement. By the first player's position strategy $U = U(t, x)$ we mean a mapping which associates a set $U(t, x) \subset P$ with each game position $\{t, x\}$. Any absolutely continuous function $x[t] = x[t; t_0, x_0, U]$, being a uniform limit of the Euler polygonal lines $x_\Delta[t] = x_\Delta[t; t_0, x_0, U]$ which satisfy the following condition

$$\begin{gathered} \frac{dx_\Delta}{dt} \in A(t)x_\Delta + F(t, u[\tau_i]) \\ x_\Delta[t_0] = x_0 \end{gathered} \tag{1.2}$$

is called a motion of system (1.1) generated by strategy U. Here

$$\tau_i \leqslant t < \tau_{i+1}, \qquad \tau_{i+1} - \tau_i \leqslant \Delta, \qquad \Delta \to 0$$
$$u[\tau_i] \in U(\tau_i, x_\Delta[\tau_i]), \quad F(t, u) = \operatorname{co}\{f(t, u, v);\ v \in Q\}$$

In what follows co N denotes the closed convex envelope of set N. It is well known that the family of motions $x[t; t_0, x_0, U]$ is a nonempty self-compact set. We pose the following problem.

Problem 1.1. Suppose that a finite-dimensional norm ρ has been defined in the space $\{x\}_m$. It is required to construct the first player's position strategy U , leading any motion $x[t] = x[t; t_0, x_0, U]$ onto set M by the instant ϑ. This signifies that the estimate

$$\min_t \rho(\{x[t]\}_m, M) = 0, \quad t_0 \leqslant t \leqslant \vartheta$$

is fulfilled for each motion $x[t] = x[t; t_0, x_0, U]$.

Let us write out the basic elements of the program construction [1, 2] to be used in solving the problem posed above. By $X[\tau, t]$ we denote the fundamental matrix of the homogeneous equation

$$dx/d\tau = A(\tau)x$$

and we introduce the following two functions:

$$\varphi(t, x, l, \tau) = l'\{X[\tau, t]x\}_m - \max_{m\in M} l'm + \tag{1.3}$$
$$\int_t^\tau [\min_{u\in P}\max_{v\in Q} l'\{X[\tau, \xi]f(\xi, u, v)\}_m]\,d\xi$$
$$\varepsilon(t, x, \tau) = \max_{\rho^*(l)=1} \varphi(t, x, l, \tau) \tag{1.4}$$

Here l is an m-dimensional vector, the prime denotes transposition, ρ^* is the norm in the space adjoint to $\{x\}_m$ with metric ρ. The quantity $\varepsilon(t, x, \tau)$ $(\varepsilon(t, x, \tau) \geqslant 0)$ is the program minimax of the distance [2] from the vector $\{x[\tau]\}_m$ to set M at the instant τ if an auxiliary program game started from position $\{t, x\}$. We define the function

$$\varepsilon(t, x) = \min_\tau \varepsilon(t, x, \tau), \quad t \leqslant \tau \leqslant \vartheta \tag{1.5}$$

We shall subsequently use the following sets:

$$L(t, x, \tau) = \{l_0:\ \rho^*(l_0) = 1;\ \varphi(t, x, l_0, \tau) = \varepsilon(t, x, \tau)\} \tag{1.6}$$

$$T(t, x) = \{\tau_0:\ \tau_0 \in [t, \vartheta],\ \varepsilon(t, x, \tau_0) = \varepsilon(t, x)\} \tag{1.7}$$

where the functions $\varphi(t, x, l, \tau)$, $\varepsilon(t, x, \tau)$, $\varepsilon(t, x)$ are given by relations (1.3)–(1.5), respectively. We assume that the following condition is fulfilled.

Condition 1.1. The inequality

$$\min_{f\in F}\min_{\tau_0\in T}\max_{l_0\in L} \psi(t, l_0, \tau_0, f) \leqslant 0 \tag{1.8}$$

is valid in the region $\{t, x\}$, where $\varepsilon(t\ x) > 0$ for any function $v_u = v(u)$ mapping set P into set Q .

Here
$$\psi(t, l, \tau, f) = l'\{X[\tau, t]f\}_m - \min_{u\in P}\max_{v\in Q} l'\{X[\tau, t]f(t, u, v)\}_m \tag{1.9}$$

$$F = F(t, v_u) = \operatorname{co}\{f(t, u, v(u));\ u \in P\} \tag{1.10}$$
$$L = L(t, x, \tau), \qquad T = T(t, x)$$

From [2] it follows that if the sets $L(t, x, \tau_0)$, $T(t, x)$ consist of the single values

$l_0 = l_0(t, x, \tau_0)$, $\tau_0 = \tau_0(t, x)$ when $\varepsilon(t, x) > 0$, then the first player's strategy U is constructed in effective form, guaranteeing the estimate

$$\min_t \rho(\{x[t]\}_m, M) \leqslant d \tag{1.11}$$

where

$$t_0 \leqslant t \leqslant \vartheta \qquad \alpha = \{0, \varepsilon(t_0, x_0)\}$$

for any motion $x[t] = x[t; t_0, x_0, U]$. In the given case the motions of system (1.1) can be defined as the solutions of the corresponding contingency equations.

In this paper we do not assume the uniqueness of the extremal elements $l_0 = l_0(t, x, \tau_0)$, $\tau_0 = \tau_0(t, x)$ in the region $\varepsilon(t, x) > 0$. Instead of this we have presented the more general Condition 1.1 and with its fulfillment we have succeeded in determining the position strategy U ensuring estimate (1.11). Generally speaking, however, we have not succeeded in constructing the first player's strategy U in an effective manner.

2. We give the solution of the encounter game problem. We construct a certain system of nonempty closed sets $W_\alpha(t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$. Here the vector $w \in W_\alpha(t, \vartheta)$ if and only if the inequality

$$\varepsilon(t, w) \leqslant \alpha \qquad (\alpha = \max\{0, \varepsilon(t_0, x_0)\}) \tag{2.1}$$

is valid. The sets $W_\alpha(t, \vartheta)$ are obviously closed since the function $\varepsilon(t, w)$ in (1.5) is continuous with respect to $\{t, w\}$. Furthermore, these sets are not empty because the inclusion

$$M^{(\alpha)} \subset W_\alpha(t, \vartheta)$$
$$M^{(\alpha)} = \{w: \ \rho(\{w\}_m, M) \leqslant \alpha\}$$

always holds. We present the definition of a u-stable system of sets.

Definition 2.1. A system of sets $W_\alpha(t, \vartheta)$ is said to be u-stable relative to $M^{(\alpha)}$ if arbitrary values $t_* \in [t_0, \vartheta]$, $w_* \in W_\alpha(t_*, \vartheta)$, $\delta \in [0, \vartheta - t_*)$, for any function v_u^* mapping set P into set Q, we can find at least one motion $y^*[t] = y^*[t; t_*, w_*, v_u^*]$ satisfying the equation

$$\frac{dy^*}{dt} \in A(t) y^* + F(t, v_u^*)$$
$$y^*[t_*] = w_* \tag{2.2}$$

and for which one of the two inclusions is fulfilled: either

$$y^*[t_* + \delta] \in W_\alpha(t_*, + \delta, \vartheta)$$

or $y^*[\eta] \in M^{(\alpha)}$ for some $\eta \in [t_*, t_* + \delta]$. According to [3] we can construct a strategy $U^{(e)}$ extremal on the system of sets $W_\alpha(t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$. If the system of sets $W_\alpha(t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$ is u-stable relative to $M^{(\alpha)}$, then the strategy $U^{(e)}$ extremal on these sets guarantees estimate (1.11) for any motion $x[t] = x[t; t_0, x_0, U^{(e)}]$ of system (1.1).

3. Let us show that when Condition 1.1 is fulfilled the system of sets $W_\alpha(t, \vartheta)$ is u-stable relative to set $M^{(\alpha)}$. We consider the motions $y[t] = y[t; t_*, y_*, f]$ satisfying on the interval $[t_*, t_* + \delta]$ the equation

$$dy/dt = A(t) y + f, \qquad y[t_*] = y_* \tag{3.1}$$

where the vector $f \in F(t_*, v_u)$ of (1.10) is chosen constant. The proof of the assertion on the stability of sets $W_\alpha(t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$ ensues from the following lemmas.

Lemma 3.1. Let the game position $\{t_*, y_*\}$ be such that the quantity $\varepsilon(t_*, y_*)$ of (1.5) is strictly positive and the number t_* does not belong to the set $T(t_*, y_*)$ of (1.7). When Condition 1.1 is fulfilled, for any function $v_u = v(u)$ mapping P into Q and for a number $\beta > 0$ we can find a vector $f^* \in F(t_*, v_u)$ of (1.10) and a number $\delta > 0$ such that the estimate

$$\varepsilon(t, y_*[t]) \leqslant \varepsilon(t_*, y_*) + \beta(t - t_*) / 2$$

is fulfilled for the motion $y_*[t] = y_*[t; t_*, y_*, f^*]$.

Proof. We compute the total derivative of the function

$$\varphi[t, l, \tau] = \varphi(t, y[t], l, \tau)$$

along any motion $y[t] = y[t; t_*, y_*, f]$ of system (3.1). From (1.3), (1.9) we obtain

$$d\varphi[t, l, \tau]/d\tau = \psi(t, l, \tau, f) \tag{3.2}$$

By the lemma's hypotheses the quantity $\varepsilon(t_*, y_*)$ is strictly positive, therefore, inequality (1.8) holds for any function $v_u = v(u)$ mapping set P into Q. Consequently, we can find a number $\tau_0^* \in T(t_*, y_*)$ of (1.7) $(\tau_0^* > t_*)$ for which the estimate

$$\min_{f \in F} \max_{l_0 \in L} \psi(t_*, l_0, \tau_0^*, f) \leqslant 0 \tag{3.3}$$

is fulfilled, where $F = F(t_*, v_u)$ of (1.10), $L = L(t_*, y_*, \tau_0^*)$ of (1.6).

The set $L(t, y, \tau)$ is semicontinuous in $\{t, y, \tau\}$, therefore, for a number $\beta > 0$ we can find $\zeta > 0$ satisfying the relation

$$\min_{f \in F} \max_{l \in L(\zeta)} \psi(t, l, \tau_0^*, f) \leqslant \beta/2 \tag{3.4}$$

Here

$$\begin{gathered} F = F(t, v_u) \\ L(\zeta) = L(t_*, y_*, \tau_0^*, \zeta) = \bigcup_{\{t, y\}} L(t, y, \tau_0^*) \\ |t - t_*| \leqslant \zeta, \qquad \|y - y_*\| \leqslant \zeta \end{gathered} \tag{3.5}$$

For the chosen number $\zeta > 0$ we can select a number $\gamma > 0$ such that the inequality

$$\|y[t] - y_*\| \leqslant \zeta$$

is valid for any motion $y[t] = y[t; t_*, y_*, f]$ of (3.1), provided $|t - t_*| \leqslant \gamma$ $(\gamma \leqslant \zeta)$. We now choose the vector $f^* \in F(t_*, v_u)$ from the condition that the left-hand side of expression (3.4) is minimum. By virtue of (3.2), (3.4), for the motion $y_*[t] = y_*[t; t_*, y_*, f^*]$ of (3.1) we obtain

$$\begin{gathered} \frac{d\varphi(t, l, \tau_0^*]}{dt} \leqslant \frac{\beta}{2} \\ \varphi(t, y_*[t], l, \tau_0^*) \leqslant \varphi(t_*, y_*, l, \tau_0^*) + \beta(t - t_*) / 2 \end{gathered} \tag{3.6}$$

under the condition that $l \in L(\zeta) = L(t_*, y_*, \tau_0^*, \zeta)$ of (3.5)

$$t \in [t_*, t_* + \delta], \quad \delta = \min\{\gamma, (\tau_0^* - t_*)\}$$

The set $L(t_*, y_*, \tau_0^*, \zeta)$ contains the sets

$$L(t, y_*[t], \tau_0^*), L(t_*, y_*, \tau_0^*)$$

therefore, from the last inequality in (3.6), and also from (1.4), (1.6), we have

$$\varepsilon(t, y_*[t], \tau_0^*) \leqslant \varepsilon(t_*, y_*, \tau_0^*) + \beta(t - t_*)/2$$

The estimate

$$\varepsilon(t, y_*[t]) \leqslant \varepsilon(t_*, y_*) + \beta(t - t_*)/2$$

follows from this and from the inclusion $\tau_0^* \in T(t_*, y_*)$, which proves Lemma 3.1.

Let $v_u = v(u)$ be an arbitrary single-valued function mapping the elements of set P into the elements of set Q. We consider the motions satisfying the differential inclusion

$$dy_\beta/dt \in A(t)\, y_\beta + F^{(\beta)}(t, v_u) \qquad (3.7)$$

$$y_\beta[t_*] = y_*$$

Here $F^{(\beta)}(t, v_u)$ is the Euclidean β-neighborhood of set $F(t, v_u)$ of (1.10).

Lemma 3.2. Let Condition 1.1 be fulfilled. Then, for any number $\beta > 0$ we can find a number η_β° $(t_* \leqslant \eta_\beta^\circ \leqslant \vartheta)$ and a motion $y_\beta^\circ[t] = y_\beta^\circ[t; t_*, y_*, v_u]$, of system (3.7), for which the relation

$$\varepsilon(t, y_\beta^\circ[t]) \leqslant \varepsilon^\circ(t_*, y_*) + \beta(t - t_*)$$

is fulfilled, where

$$t_0 \leqslant t \leqslant \eta_\beta^\circ, \quad \varepsilon^\circ(t_*, y_*) = \max\{0, \varepsilon(t_*, y_*)\}$$

$$\rho(\{y_\beta^\circ[\eta_\beta^\circ]\}_m, M) \leqslant \varepsilon^\circ(t_*, y_*) + \beta(\eta_\beta^\circ - t_*)$$

In fact, because function (1.5) is continuous, we can find the largest number $\eta = \eta(y_\beta[\cdot])$ $(\eta \leqslant \vartheta)$, for which the inequality

$$\varepsilon(t, y_\beta[t]) \leqslant \varepsilon^\circ(t_*, y_*) + \beta(t - t_*) \qquad (3.8)$$

holds for $t_* \leqslant t \leqslant \eta$, $y_\beta[t] = y_\beta[t; t_*, y_*, v_u]$ from (3.7). By η_β° we denote the upper bound of the numbers $\eta = \eta(y_\beta[\cdot])$ in (3.8) over all possible motions of system (3.7), i.e.

$$\eta_\beta^\circ = \sup_{y_\beta[\cdot]} \eta(y_\beta[\cdot]) \qquad (3.9)$$

By virtue of the compactness of the solutions of Eq. (3.7), this upper bound is reached on some motion $y_\beta^\circ[t] = y_\beta^\circ[t; t_*, y_*, v_u]$. Let us show that the number η_β° of (3.9) belongs to the set $T(\eta_\beta^\circ, y_\beta^\circ[\eta_\beta^\circ])$ of (1.7). Assume that the number η_β° does not belong to this set. Then, obviously, $\eta_\beta^\circ < \vartheta$. We consider all solutions of (3.7), $\{y_\beta^*[t] = y_\beta^*[t; t_*, y_*, v_u]\}$, satisfying the equality

$$y_\beta^*[t] = y_\beta^\circ[t] \qquad (t_* \leqslant t \leqslant \eta_\beta^\circ)$$

By virtue of the choice of number η_β°, the relation

$$\max_t \{\varepsilon(t, y_\beta^*[t]) - \beta(t - t_*)\} > \varepsilon^\circ(t_*, y_*) \qquad t_* \leqslant t \leqslant \eta_\beta^\circ + \delta, \qquad (3.10)$$

$$\varepsilon^\circ(t_*, y_*) = \max\{0, \varepsilon(t_*, y_*)\}$$

is fulfilled for each motion and for any number $\delta \in (0, \vartheta - \eta_\beta^*]$. Here

$$\varepsilon(\eta_\beta^\circ, y_\beta^\circ[\eta_\beta^\circ]) = \varepsilon^\circ(t_*, y_*) + \beta(\eta_\beta^\circ - t_*) \qquad (3.11)$$

On the other hand, from (3.11) and Lemma 3.1 it follows that for the number $\beta > 0$ we can find a vector $f^* \in F(\eta_\beta^\circ, v_u)$ and a number $\delta^\circ > 0$ ensuring for the motion $y_*[t] = y_*[t;\ \eta_\beta^\circ,\ y_\beta^\circ[\eta_\beta^\circ], f^*]$ the inequality and the inclusion

$$\varepsilon(t, y_*[t]) \leqslant \varepsilon(\eta_\beta^\circ, y_\beta^\circ[\eta_\beta^\circ]) + \beta(t - t_*)/2 \tag{3.12}$$

$$f^* \in F^{(\beta)}(t, v_u)$$

for $\eta_\beta^\circ \leqslant t \leqslant \eta_\beta^\circ + \delta^\circ$. Thus, we have constructed the motion $y_\beta^*[t] = y_\beta^*[t;\ t_*,\ y_*,\ v_u]$ $(y_\beta^*[t] = y_\beta^\circ[t]$ for $t_* \leqslant t \leqslant \eta_\beta^\circ$, $y_\beta^*[t] = y_*[t]$ for $\eta_\beta^\circ \leqslant t \leqslant \eta_\beta^\circ + \delta^\circ$ satisfying the estimate

$$\varepsilon(t, y_\beta^*[t]) \leqslant \varepsilon^\circ(t_*, y_*) + \beta(t - t_*) \tag{3.13}$$

This last relation follows immediately from (3.11),(3.12). Inequalities (3.10) and (3.13) are contradictory and, thus, $\eta_\beta^\circ \in T(\eta_\beta^\circ, y_\beta^\circ[\eta_\beta^\circ])$ of (1.7). But then from the definitions of the function $\varepsilon(t, y)$ of (1.5) and of the sets $T(t, y)$ of (1.7) we obtain

$$\rho(\{y_\beta^\circ[\eta_\beta^\circ]\}_m, M) \leqslant \varepsilon^\circ(t_*, y_*) + \beta(\eta^\circ_\beta - t_*)$$

$$\varepsilon(t, y_\beta^\circ[t]) \leqslant \varepsilon^\circ(t_*, y_*) + \beta(t - t^*)$$

for $t_* \leqslant t \leqslant \eta_\beta^\circ$. These relations prove Lemma 3.2.

We now show that the sets $W_\alpha(t, \vartheta)$ of (2.1) are u-stable relative to the set $M^{(\alpha)} = \{x : \rho(\{x\}_m, M) \leqslant \alpha\}$.

Lemma 3.3. Let Condition 1.1 be fulfilled; then the sets $W_\alpha(t, \vartheta)$ are u-stable relative to $M^{(\alpha)}$.

Proof. We choose the values

$$t_* \in [t_0, \vartheta], \quad w_* \in W_\alpha(t_*, \vartheta), \delta \in [0, \vartheta - t_*]$$

arbitrarily and we specify the function $v_u = v(u)$ mapping set P into Q. Let $\beta_n > 0$ be a sequence of numbers converging to zero. According to Lemma 3.2, for each number n we can find a number η_n $(t_* \leqslant \eta_n \leqslant \vartheta)$ and a motion $y_n[t] = y_n[t; t_*, w_*, v_u]$, which satisfy the relations

$$\begin{aligned} & dy_n / dt \in A(t)y_n + F^{(\beta_n)}(t, v_u);\ y_n[t_*] = w_* \\ & \varepsilon(t, y_n[t]) \leqslant \varepsilon^\circ(t_*, w_*) + \beta_n(t - t_*) \\ & \rho(\{y_n[\eta_n]\}_m, M) \leqslant \varepsilon^\circ(t_*, w_*) + \beta_n(\eta_n - t_*) \\ & t_* \leqslant t \leqslant \eta_n, \qquad \eta_n \leqslant \vartheta \end{aligned} \tag{3.14}$$

From the sequence of motions $y_n[t; t_*, w_*, v_u]$ we can choose a subsequence $y_{n_k}[t; t_*, w_*, v_u]$ converging uniformly to some function $y_*[t; t_*, w_*, v_u]$ which, obviously, is a motion of system (2.2). Furthermore, from the subsequence of numbers η_{n_k} we can choose a subsequence converging to some instant η_*. Thus, from (3.14) we obtain

$$\varepsilon(t, y_*[t]) \leqslant \varepsilon^\circ(t_*, w_*) \leqslant \alpha$$

$$t_0 \leqslant t \leqslant \eta_*, \qquad \varepsilon^\circ(t_*, w_*) = \max\{0, \varepsilon(t^*, w^*)\}$$

for the limit motion $y_*[t] = y_*[t; t_*, w_*, v_u]$. The u-stability of sets $W_\alpha(t, \vartheta)$ $(t_0 \leqslant t \leqslant \vartheta)$ follows directly from this and from (2.1), which proves Lemma 3.3.

From Lemma 3.3 we finally obtain the following assertion.

Theorem 3.1. When Condition 1.1 is fulfilled, the first player's position strategy U exists, which guarantees the estimate

$$\min_t \rho(\{x[t]\}_m, M) \leqslant \alpha$$
$$t_0 \leqslant t \leqslant \vartheta, \quad \alpha = \max\{0, \varepsilon(t_0, x_0)\}$$

for any motion $x[t] = x[t; t_0, x_0, U]$.

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ON A GENERALIZATION OF THE THEORY OF ERROR ACCUMULATION

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I. Ia. KRICHEVSKII and G. M. ULANOV
(Khar'kov and Moscow)

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We pose the problem of error accumulation in linear systems on a finite time interval under three constraints of the perturbation function and its lower derivatives. We have shown that the largest error in the system is realized in the class of piecewise-quadratic functions possessing certain limit properties on the set of switching points of the system's impulse transient response and of the maximizing external influence. Schemes are obtained for the effective solution of the problem, based on a combination of Bellman's optimality principle and of analytic information on the extremal properties of the external influences. The present paper is a development of [1 – 3].

1. Statement of the problem. Let the error in the kth system coordinate x_k $(k = 1, \ldots, l)$, caused by a perturbing action $f(t)$, be the solution of the differential equation

$$\frac{d^n x_k}{dt^n} = \sum_{i=0}^{n-1} A_i(t) \frac{d^i(x_k)}{dt^i} + f(t) \tag{1.1}$$

in which the coefficients $A_i(t)$ $(i = 0, \ldots, n-1)$ are continuous functions of